# 3.3 Truth Tables for the Conditional and Biconditional

Math 120

Math for General Education

Michael Orr

## Truth Table - Conditional

"If you get an A in this class, then I will buy you a car."

p: You get an A.

q: I buy you a car.

False only if the promise is broken.

| | p | q | p → q |
|---|---|---|---|
| Case 1 | T | T | T |
| Case 2 | T | F | F |
| Case 3 | F | T | T |
| Case 4 | F | F | T |

The conditional statement p → q is true in every case except when p is true and q is false.

# Truth Table – Conditional

$\sim p \rightarrow \sim q$

| | 1 | 2 | 3 | 5 | 4 |
|---|---|---|---|---|---|
| | p | q | ~p | → | ~q |
| Case 1 | T | T | F | T | F |
| Case 2 | T | F | F | T | T |
| Case 3 | F | T | T | F | F |
| Case 4 | F | F | T | T | T |

*Cases 1, 2, and 4 the statement is true.

*Cases 3 & 5 – 8 produced true answers.

# Truth Table - Conditional

$p \rightarrow (\sim q \wedge r)$

| Clmn# | 1 | 2 | 3 | 4 | 8 | 6 | 7 | 5 |
|---|---|---|---|---|---|---|---|---|
| Case# | p | q | r | p | → | (~q | ∧ | r) |
| 1 | T | T | T | T | F | F | F | T |
| 2 | T | T | F | T | F | F | F | F |
| 3 | T | F | T | T | T | T | T | T |
| 4 | T | F | F | T | F | T | F | F |
| 5 | F | T | T | F | T | F | F | T |
| 6 | F | T | F | F | T | F | F | F |
| 7 | F | F | T | F | T | T | T | T |
| 8 | F | F | F | F | T | T | F | F |

# Truth Table - Biconditional

- **Remember:** p ←→ q **reads** "p if and only if q"
- **Means:** **(p → q) Λ (q→ p)**
- Truth table for p←→q **will be same as for** (p→q) Λ (q → p).

# Truth Table for: (p → q) Λ (q→ p)

| Clmn# | 1 | 2 | 3 | 5 | 4 | 9 | 6 | 8 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| Case # | p | q | (p | → | q) | Λ | (q | → | p) |
| 1 | T | T | T | T | T | T | T | T | T |
| 2 | T | F | T | F | F | F | F | T | T |
| 3 | F | T | F | T | T | F | T | F | F |
| 4 | F | F | F | T | F | T | F | T | F |

# Therefore:

| p | q | p←→q |
|---|---|---|
| T | T | *T* |
| T | F | *F* |
| F | T | *F* |
| F | F | *T* |

**p ←→ q is true only when p and q have the same truth value (both are T or F).**

*Cases 2 - 5 produced true answers.

# Truth Table - Biconditional

p ←→(q → ~r)

| Clmn# | 1 | 2 | 3 | 4 | 8 | 5 | 7 | 6 |
|---|---|---|---|---|---|---|---|---|
| Case# | p | q | r | p | ←→ | (q | → | ~r) |
| 1 | T | T | T | T | F | T | F | F |
| 2 | T | T | F | T | T | T | T | T |
| 3 | T | F | T | T | T | F | T | F |
| 4 | T | F | F | T | T | F | T | T |
| 5 | F | T | T | F | T | T | F | F |
| 6 | F | T | F | F | F | T | T | T |
| 7 | F | F | T | F | F | F | T | F |
| 8 | F | F | F | F | F | F | T | T |

## Determining the Truth Value of a Compound Statement

- $(q \leftrightarrow r) \rightarrow (\sim p \wedge r)$
- Evaluate when p is T, q is F, and r is T

$$(q \leftrightarrow r) \rightarrow (\sim p \wedge r)$$

$$(F \leftrightarrow T) \rightarrow (F \wedge T)$$

$$F \quad \rightarrow \quad F$$

$$T$$

## Determining the Truth Value of a Compound Statement

Northwestern University is in Illinois and Marquette Univ. is in Alaska, if and only if Purdue University is in Alabama.

Let: p: Northwestern Univ. is in Illinois
q: Marquette Univ. is in Alaska
r: Purdue Univ. is in Alabama

$$(p \wedge q) \leftrightarrow r$$

$$(T \wedge F) \leftrightarrow F$$

$$F \leftrightarrow F$$

$$T$$

# Self-Contradictions, Tautologies, and Implications

- Self-Contradiction – compound statement that is always false.

$(p \leftrightarrow q) \wedge (p \leftrightarrow \sim q)$

| p | q | (p | ↔ | q) | ∧ | (p | ↔ | ~q) |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | T | F | F |
| T | F | T | F | F | F | T | T | T |
| F | T | F | F | T | F | F | T | F |
| F | F | F | T | F | F | F | F | T |
| | | 1 | 5 | 2 | 7 | 3 | 6 | 4 |

# Self-Contradictions, Tautologies, and Implications (cont.)

- Tautology – compound statement that is always true.

$(p \wedge q) \rightarrow (p \vee r)$

| p | q | r | (p ∧ q) | → | (p ∨ r) |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | T | T |
| T | F | T | F | T | T |
| T | F | F | F | T | T |
| F | T | T | F | T | T |
| F | T | F | F | T | F |
| F | F | T | F | T | T |
| F | F | F | F | T | F |
| | | | 1 | 3 | 2 |

## Self-Contradictions, Tautologies, and Implications (cont.)

- Conditional statements that are tautologies are called ***implications***.
- From the last example, we can say that **p Λ q implies p V r**.

## Practice Problems

- IN CLASS
  - Page 133
    - #14, 34
- HOMEWORK
  - Pages 133 – 135
    - #1 – 6 all, 9 – 66 multiples of 3